BIRZEIT UNIVERSITY

Faculty of Engineering and Technology
Department of Electrical and Computer Engineering
Modern Communication Systems ENEE 3306
Instructor: Dr. Wael Hashlamoun
Midterm Exam
Second Semester 2018-2019

Date: Sunday April 7, 2019 — Time: 75 minutes
Name: — Student #:

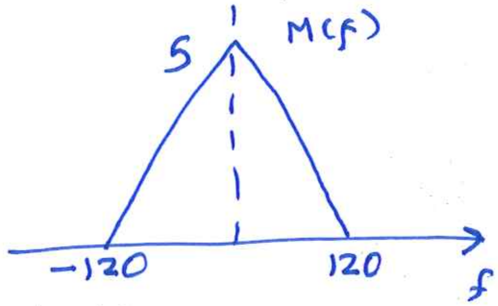
Opening Remarks:

- Calculators are allowed, however, mobile phones, books, notes, formula sheets, and other aids are not allowed.
- You are required to show all your work and provide the necessary explanations everywhere to get full credit.

Problem 1: 25 Points

Consider the signal $m(t)$, with spectrum $M(f)$, given by,

$$M(f) = \begin{cases} 5 - f/120\ , & 0 < f \le 120 \\ 5 + f/120, & -120 \le f \le 0 \\ 0 & , \textit{otherwise} \end{cases}$$



This signal is multiplied by $c(t)$ to get signal $m_s(t)$, where

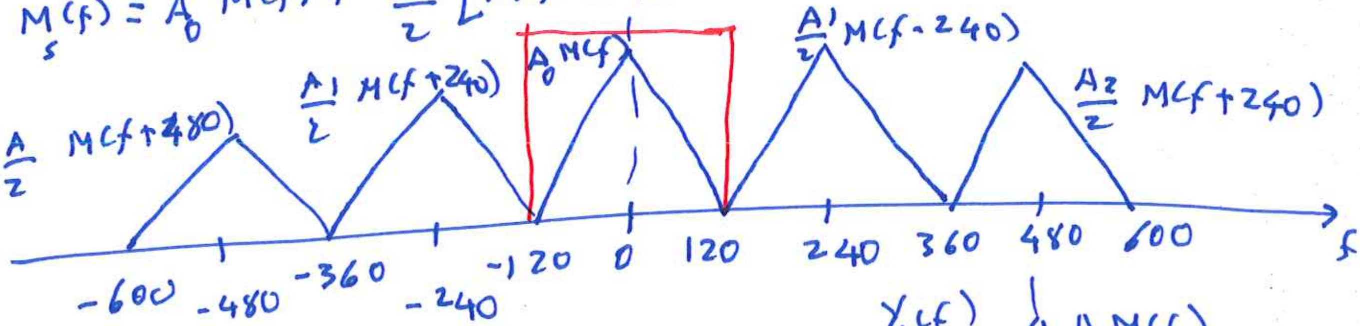
$$c(t) = A_0 + A_1 cos(2\pi(240)t) + A_2 cos(2\pi(480)t)$$

a. What is the absolute bandwidth of $m(t)$?
b. Sketch $M_s(f)$, the spectrum of $m_s(t)$.
c. If $m_s(t)$ is passed through an ideal low pass filter with a bandwidth of 120 Hz to produce an output $y_1(t)$. Sketch $Y_1(f)$.
d. Is $y_1(t)$ *proportional to* $m(t)$? Explain why.

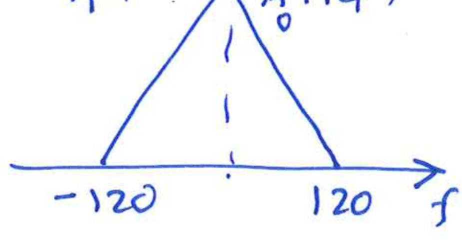
a. $B.W = (120 - 0) = 120\ Hz$

b. $m_s(t) = m(t)\,c(t) = m(t)[A_0 + A_1 \cos 2\pi(240)t + A_2 \cos 2\pi(480)t]$

$M_s(f) = A_0 M(f) + \frac{A_1}{2}[M(f-240) + M(f+240)] + \frac{A_2}{2}[M(f-480) + M(f+480)]$



$Y_1(f) = M_s(f)\,H(f) = A_0 M(f)$



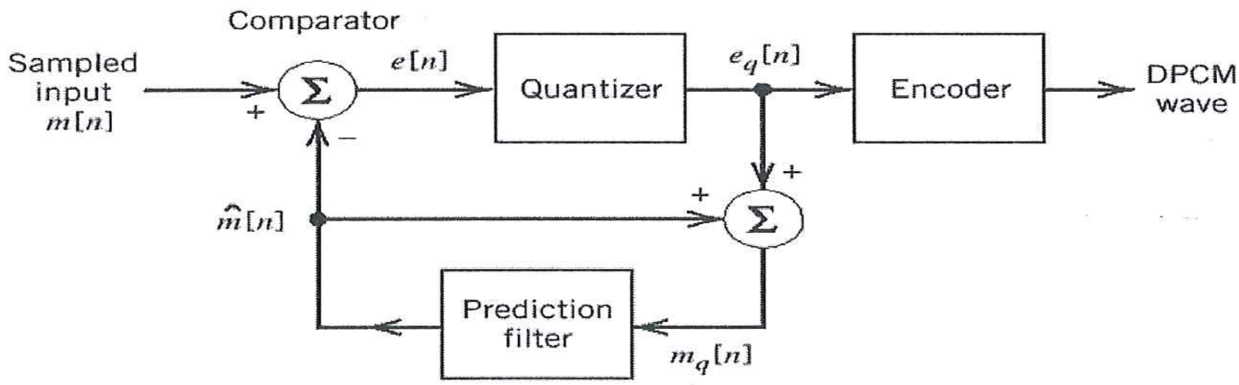
$Y_1(f) \propto M(f)$
$\Rightarrow y_1(t) \propto m(t)$
$\Rightarrow$ No distortion

Problem 2: 25 Points

Consider a differential pulse code modulator similar to the one shown in the figure below. A signal $m(t)$ with bandwidth 400 Hz is sampled at its Nyquist rate. The error $e(t)$ is applied to a 16-level uniform quantizer. The prediction is made based only on the previous sample, i.e., $\hat{m}(nT_s) = w_1 m((n-1)T_s)$. The autocorrelation function of $m(t)$ is given by

$$R_m(\tau) = \begin{cases} 3 + \left(\frac{\tau}{3T_s}\right), -3T_s \le \tau \le 0 \\ 3 - \left(\frac{\tau}{3T_s}\right), 0 \le \tau \le 3T_s \\ 0, \quad otherwise \end{cases}$$

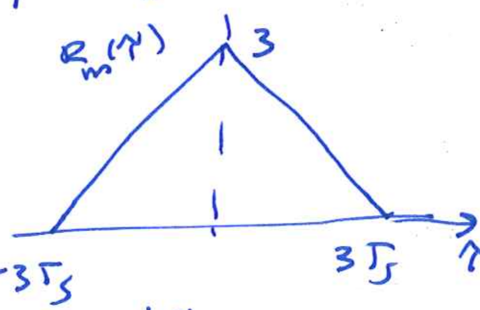
a. Find w_1 that minimizes the mean square error between the sample and its predicted value.
b. Find the sampling frequency.
c. Find the data rate in bits/sec.



$\hat{x}(n) = w_1 x(n-1) \Rightarrow \epsilon = E\{[x(n) - \hat{x}(n)]^2\}$

11 a. $R w = R_1$ or $R_x(0) w_1 = R_x(T_s)$

$\Rightarrow w_1 = \frac{R_x(T_s)}{R_x(0)}$



$w_1 = \frac{3 - \left(\frac{T_s}{3T_s}\right)}{3}$

$= \frac{3 - 1/3}{3} = \frac{8/3}{3} = \frac{8}{9} = 8/9$

7 b. $f_s = 2W = 2(400) = 800$ samples/sec

7 c. $L = 16 \Rightarrow L = 16 = 2^r = 4$ digits $= r$

$R_b = (2W)$ samples/sec $\times$ 4 digits/sample

$= 8W$ bits/sec

$= 3200$ bits/sec

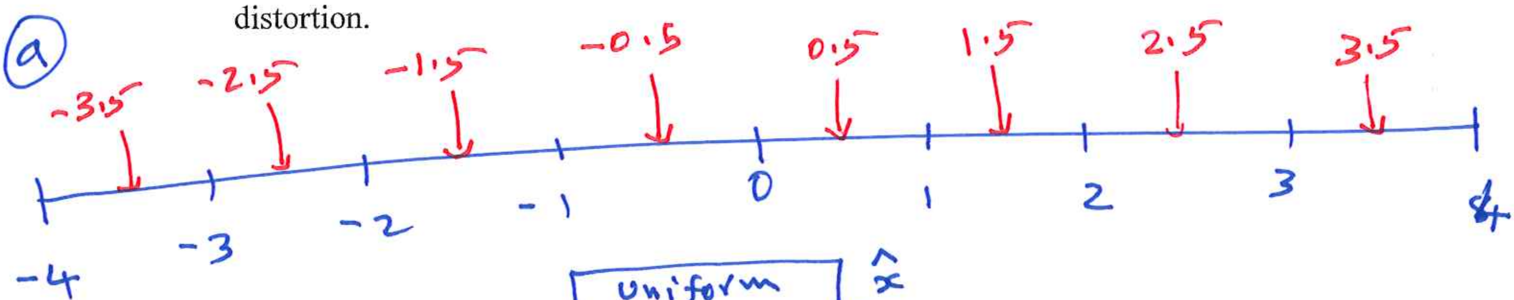
Problem 3: 25 Points

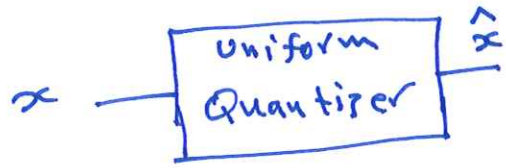
8 a. Design an 8-level uniform quantizer with a dynamic range (-4, 4) V, i.e., find the thresholds and representation values.

6 b. If a sample with a 0.15 V value is applied to the uniform quantizer of Part a, find the received signal value corresponding to this sample, and the amount of distortion affecting this sample.

11 c. If a sample with a 0.15 V value is applied to a μ-law companding system with $\mu = 255$ (a compressor followed by the uniform quantizer of Part a and then an expander), find the received signal value corresponding to this sample, and the amount of distortion.

a





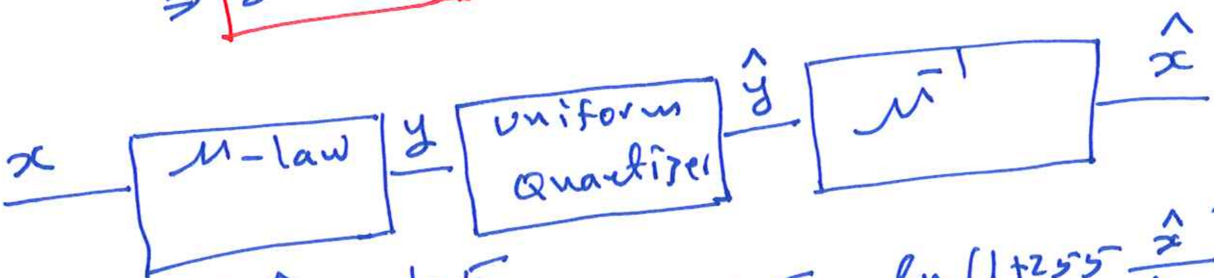
b.

$x = 0.15 \Rightarrow \hat{x} = 0.5 \Rightarrow \epsilon = |x - \hat{x}| = 0.35$

c. $\frac{y}{y_{max}} = \frac{\ln(1 + 255 \frac{x}{x_{max}})}{\ln(1+255)}$; Let $x = 0.15$

$\frac{y}{4} = \frac{\ln(1 + 255 \frac{0.15}{4})}{\ln(256)} = \frac{2.357}{5.545} = 0.425$

$\Rightarrow \boxed{y = 1.7}$



$\Rightarrow \hat{y} = 1.5$

Need to solve for $\hat{x}$ $\quad \frac{1.5}{4} = \frac{\ln(1+255\frac{\hat{x}}{4})}{\ln(256)}$

$0.375 * 5.545 = \ln(1+255\frac{\hat{x}}{4}) \Rightarrow 1 + 63.75\hat{x} = e^{2.079}$

$\hat{x} = \frac{e^{2.079} - 1}{63.75} = \frac{7.999 - 1}{63.75} = 0.1097$

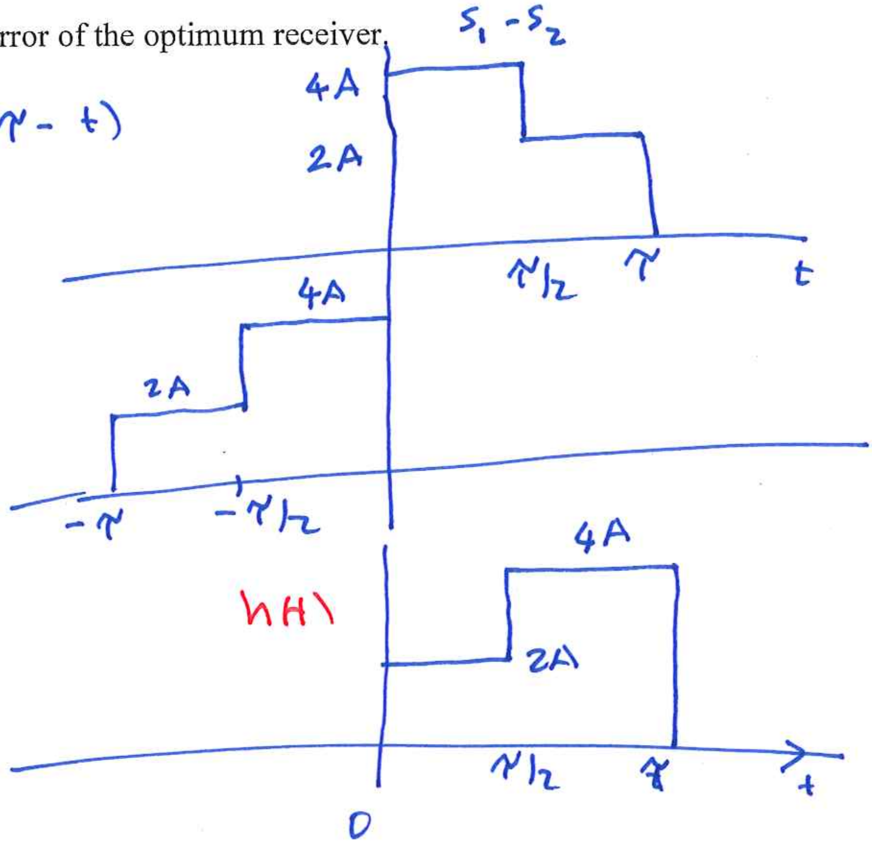
$\Rightarrow \boxed{\text{error} = |0.15 - 0.1097| = 0.04}$

Problem 4: 25 Points

A binary digital signaling scheme employs the signal $s_1(t)$ to represent digit 1 and $s_2(t) = -s_1(t)$ to represent binary digit 0, over a channel corrupted by AWGN with power spectral density $N_0/2$ W/Hz, where

$$s_1(t) = \begin{cases} 2A, & 0 \le t \le \tau/2 \\ A, & \tau/2 \le t \le \tau \\ 0, & otherwise \end{cases}$$

a. Find and sketch the optimum filter
b. Find E_1, the energy in $s_1(t)$.
c. Find the average probability of error of the optimum receiver.



a. $h(t) = s_1(\tau - t) - s_2(\tau - t)$

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b. $E_1 = \int_0^{\tau} |s_1(t)|^2 dt$

7

$= \int_0^{\tau/2} 4A^2 dt + \int_{\tau/2}^{\tau} A^2 dt$

$= 4A^2 \cdot \frac{\tau}{2} + A^2 \frac{\tau}{2}$

$E_1 = \frac{5A^2\tau}{2}$

c. $P_b = Q\left(\sqrt{\frac{\int_0^{\tau}(s_1 - s_2)^2 dt}{2N_0}}\right)$

9

$\int_0^{\tau}(s_1 - s_2)^2 dt = \int_0^{\tau/2} 16A^2 dt + \int_{\tau/2}^{\tau} 4A^2 dt = 16A^2 \frac{\tau}{2} + 4A^2 \frac{\tau}{2}$

$= 10A^2\tau$

$P_b = Q\left(\sqrt{\frac{10A^2\tau}{2N_0}}\right) = Q\left(\sqrt{\frac{5A^2\tau}{N_0}}\right)$

TABLE 4-2
Fourier Transform Pairs

Pair Number	$x(t)$	$X(f)$	Comments on Derivation
1.	$\Pi\left(\frac{t}{\tau}\right)$	$\tau \operatorname{sinc} \tau f$	Direct evaluation
2.	$2W \operatorname{sinc} 2Wt$	$\Pi\left(\frac{f}{2W}\right)$	Duality with pair 1, Example 4-7
3.	$\Lambda\left(\frac{t}{\tau}\right)$	$\tau \operatorname{sinc}^2 \tau f$	Convolution using pair 1
4.	$\exp(-\alpha t)u(t),\ \alpha > 0$	$\frac{1}{\alpha + j2\pi f}$	Direct evaluation
5.	$t\exp(-\alpha t)u(t),\ \alpha > 0$	$\frac{1}{(\alpha + j2\pi f)^2}$	Differentiation of pair 4 with respect to α
6.	$\exp(-\alpha\|t\|),\ \alpha > 0$	$\frac{2\alpha}{\alpha^2 + (2\pi f)^2}$	Direct evaluation
7.	$e^{-\pi(t/\tau)^2}$	$\tau e^{-\pi(f\tau)^2}$	Direct evaluation
8.	$\delta(t)$	1	Example 4-9
9.	1	$\delta(f)$	Duality with pair 7
10.	$\delta(t - t_0)$	$\exp(-j2\pi f t_0)$	Shift and pair 7
11.	$\exp(j2\pi f_0 t)$	$\delta(f - f_0)$	Duality with pair 9
12.	$\cos 2\pi f_0 t$	$\frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0)$	Exponential representation of cos and sin and pair 10
13.	$\sin 2\pi f_0 t$	$\frac{1}{2j}\delta(f - f_0) - \frac{1}{2j}\delta(f + f_0)$	
14.	$u(t)$	$(j2\pi f)^{-1} + \frac{1}{2}\delta(f)$	Integration and pair 7
15.	$\operatorname{sgn} t$	$(j\pi f)^{-1}$	Pair 8 and pair 13 with superposition
16.	$\frac{1}{\pi t}$	$-j \operatorname{sgn}(f)$	Duality with pair 14
17.	$\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\lambda)}{t - \lambda}\,d\lambda$	$-j \operatorname{sgn}(f)X(f)$	Convolution and pair 15
18.	$\sum_{m=-\infty}^{\infty} \delta(t - mT_s)$	$f_s \sum_{m=-\infty}^{\infty} \delta(f - mf_s),\ f_s = T_s^{-1}$	Example 4-10

TABLE A6.4 ***Trigonometric Identities***

$$\exp(\pm j\theta) = \cos\theta \pm j\sin\theta$$
$$\cos\theta = \tfrac{1}{2}[\exp(j\theta) + \exp(-j\theta)]$$
$$\sin\theta = \frac{1}{2j}[\exp(j\theta) - \exp(-j\theta)]$$
$$\sin^2\theta + \cos^2\theta = 1$$
$$\cos^2\theta - \sin^2\theta = \cos(2\theta)$$
$$\cos^2\theta = \tfrac{1}{2}[1 + \cos(2\theta)]$$
$$\sin^2\theta = \tfrac{1}{2}[1 - \cos(2\theta)]$$
$$2\sin\theta\cos\theta = \sin(2\theta)$$
$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$
$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$
$$\sin\alpha\sin\beta = \tfrac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$
$$\cos\alpha\cos\beta = \tfrac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$
$$\sin\alpha\cos\beta = \tfrac{1}{2}[\sin(\alpha - \beta) + \sin(\alpha + \beta)]$$

Table of Standard Integrals

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$

2. $\int \frac{dx}{x} = \ln|x| + C$

3. $\int e^x\,dx = e^x + C$

4. $\int \sin x\,dx = -\cos x + C$

5. $\int \cos x\,dx = \sin x + C$

6. $\int \tan x\,dx = -\ln|\cos x| + C$

7. $\int \cot x\,dx = \ln|\sin x| + C$

8. $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

9. $\int \sec^2 x\,dx = \tan x + C$

10. $\int \mathrm{cosec}^2\,x\,dx = -\cot x + C$

11. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$

12. $\int \mathrm{cosec}\,x\,dx = \ln|\mathrm{cosec}\,x - \cot x| + C$

13. $\int \sinh x\,dx = \cosh x + C$

14. $\int \cosh x\,dx = \sinh x + C$

15. $\int \tanh x\,dx = \ln\cosh x + C$

16. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C \quad (|x| < a)$

17. $\int \frac{dx}{\sqrt{x^2+a^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C = \ln\left(x + \sqrt{x^2+a^2}\right) + C'$

18. $\int \frac{dx}{\sqrt{x^2-a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + C = \ln\left(x + \sqrt{x^2-a^2}\right) + C' \quad (x > a)$

Q-Function Table

z	$Q(z)$	z	$Q(z)$
0.0	0.50000	2.0	0.02275
0.1	0.46017	2.1	0.01786
0.2	0.42074	2.2	0.01390
0.3	0.38209	2.3	0.01072
0.4	0.34458	2.4	0.00820
0.5	0.30854	2.5	0.00621
0.6	0.27425	2.6	0.00466
0.7	0.24196	2.7	0.00347
0.8	0.21186	2.8	0.00256
0.9	0.18406	2.9	0.00187
1.0	0.15866	3.0	0.00135
1.1	0.13567	3.1	0.00097
1.2	0.11507	3.2	0.00069
1.3	0.09680	3.3	0.00048
1.4	0.08076	3.4	0.00034
1.5	0.06681	3.5	0.00023
1.6	0.05480	3.6	0.00016
1.7	0.04457	3.7	0.00011
1.8	0.03593	3.8	0.00007
1.9	0.02872	3.9	0.00005